①

SIGNALS & SYSTEMS

ASSIGNMENT - 2.

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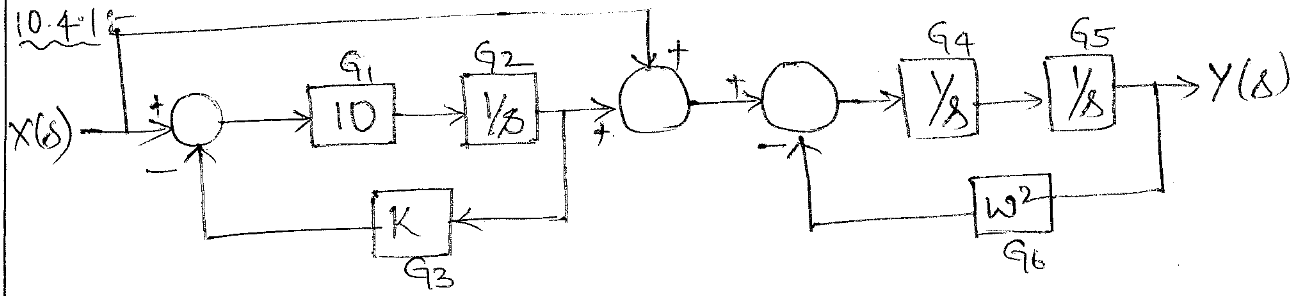
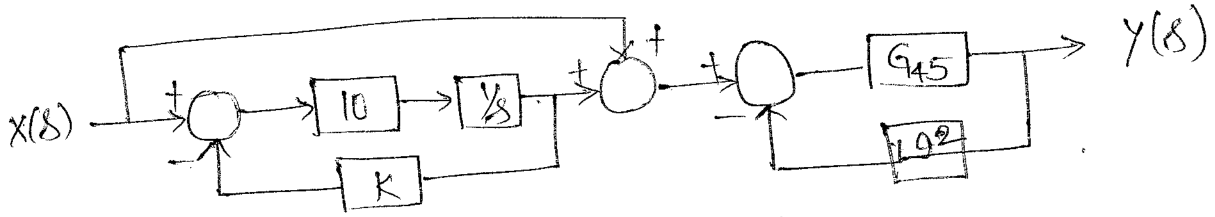
0208-5035

I. Find the transfer function in the following figures by block diagram reduction.

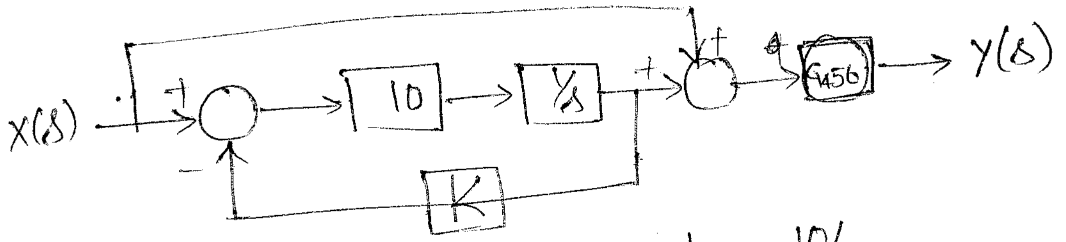
① 10.4.1



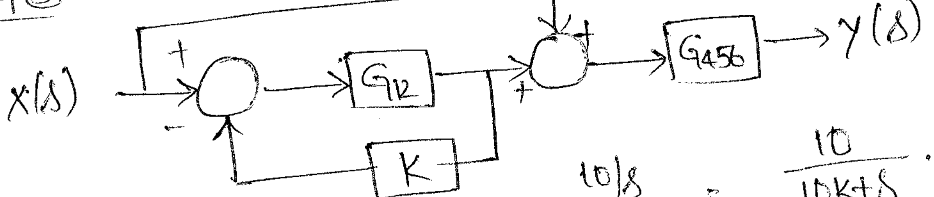
Step ①: $G_{45} = G_4 G_5 = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$



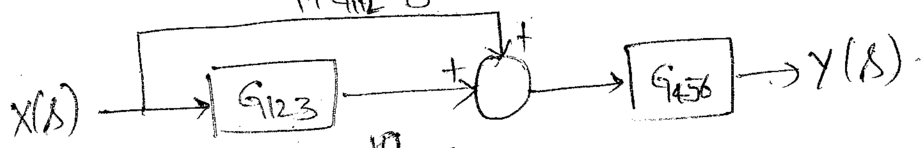
Step ②: $G_{456} = \frac{G_{45}}{1 + G_{45} \cdot G_6} = \frac{1/s^2}{1 + \frac{1}{s^2} \cdot 10^2} = \frac{1}{s^2 + 10^2}$



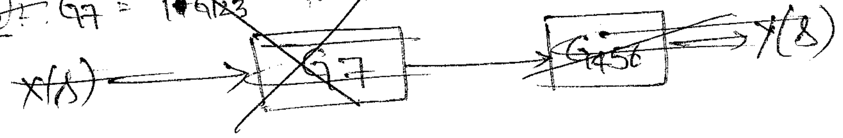
Step ③: $G_{12} = G_1 G_2 = 10 \cdot \frac{1}{s} = 10/s$.



Step ④: $G_{123} = \frac{G_{12}}{1 + G_{12} \cdot G_3} = \frac{10/s}{1 + \frac{10}{s} \cdot K} = \frac{10}{10K + s}$.



Step ⑤: $G_7 = \frac{G_{123}}{1 + G_{123}} = \frac{10}{s + 10K + 10}$



Step ⑥ : $G_{4567} = \cancel{G_{456}\, G_7} = \frac{1}{\cancel{s^2+\omega^2}} \cdot \frac{10}{\cancel{s+10k+10}}$

Step ⑤ : $[X(s) + G_{123}\,X(s)] \longrightarrow \boxed{G_{456}} \longrightarrow Y(s)$

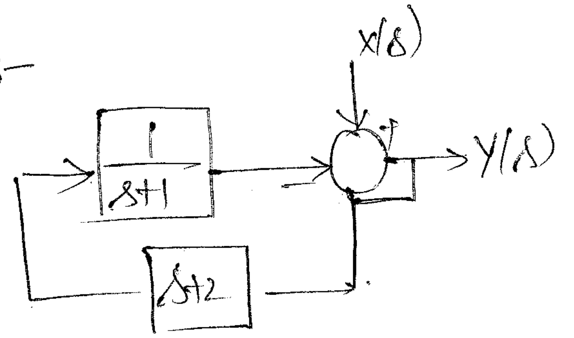
$$\therefore \frac{Y(s)}{X(s)[G_{123}+1]} = G_{456}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = (1+G_{123})\,G_{456}$$

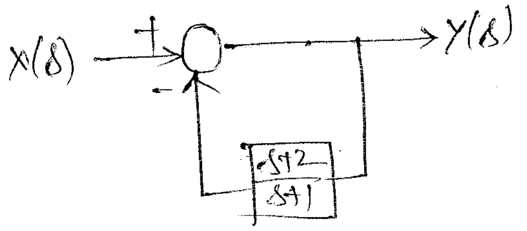
$$= \left(1+\frac{10}{s+10k}\right)\left(\frac{1}{s^2+\omega^2}\right)$$

$$= \frac{s+10k+10}{(s+10k)(s^2+\omega^2)}$$

② 10.4.3 :-



Step ① :

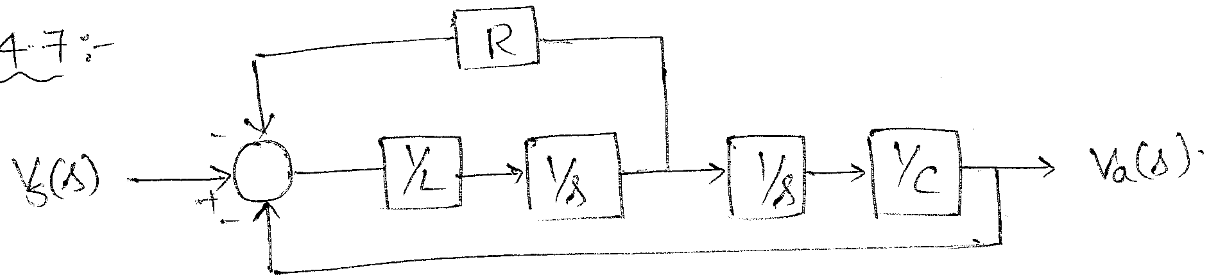


Step ② : $X(s) \longrightarrow \boxed{\frac{1}{1+\frac{s+2}{s+1}}} \longrightarrow Y(s)$

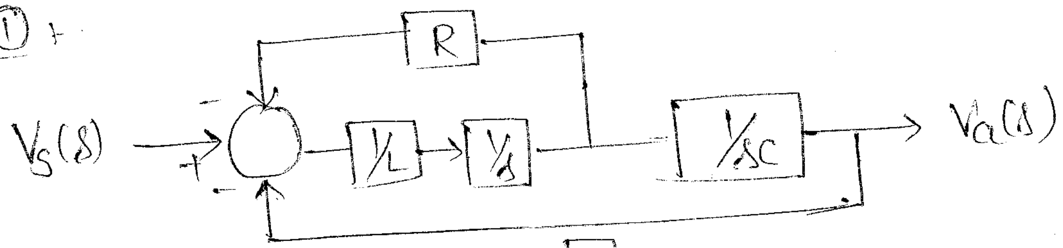
$$\therefore \frac{Y(s)}{X(s)} = \frac{1}{1+\left(\frac{s+2}{s+1}\right)\cdot 1}$$

$$= \frac{s+1}{2s+3}$$

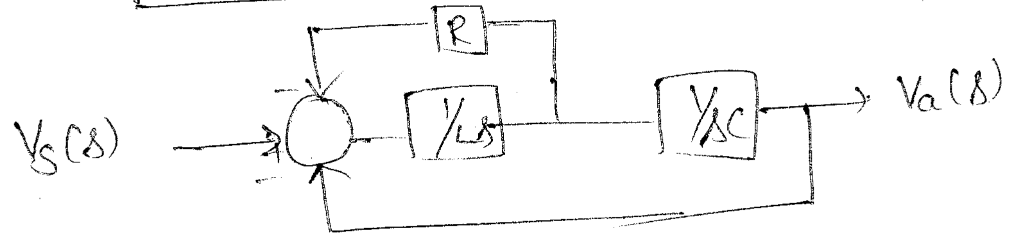
③ 10.4.7:-



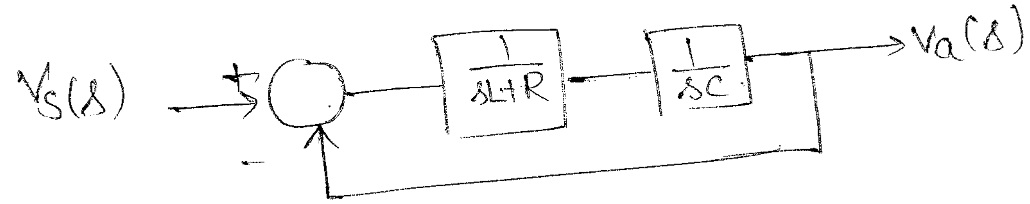
Step ①:



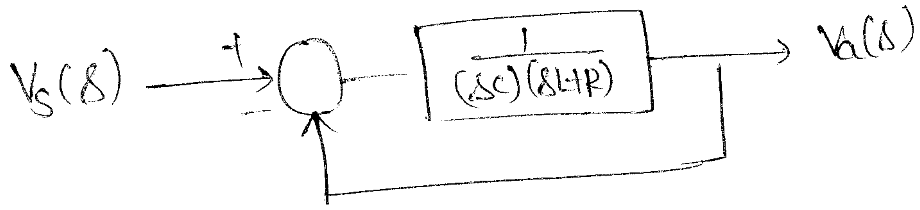
Step ②:



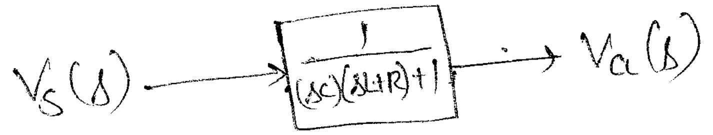
Step ③:



Step ④:



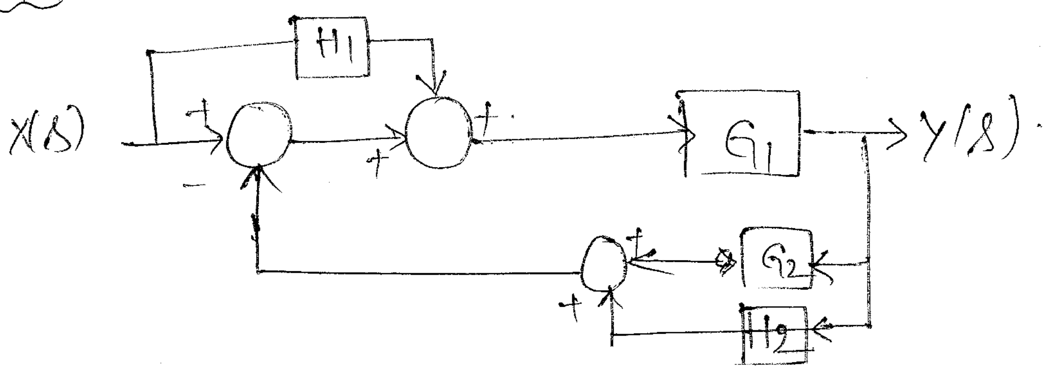
Step ⑤:



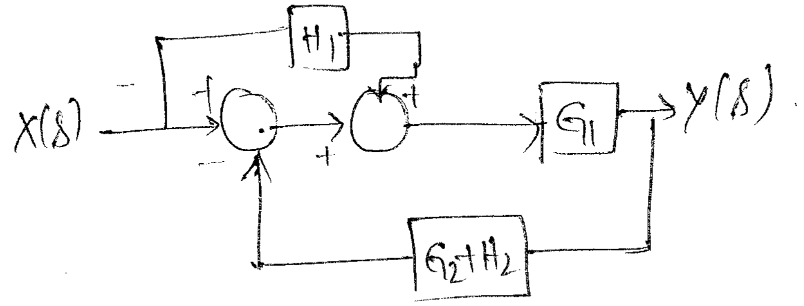
$$\frac{V_a(s)}{V_s(s)} = \frac{\frac{1}{(sC)(sL+R)}}{1+\frac{1}{(sC)(sL+R)}} = \frac{1}{(sC)(sL+R)+1}$$

$$= \frac{1}{1+sCR+s^2LC}$$

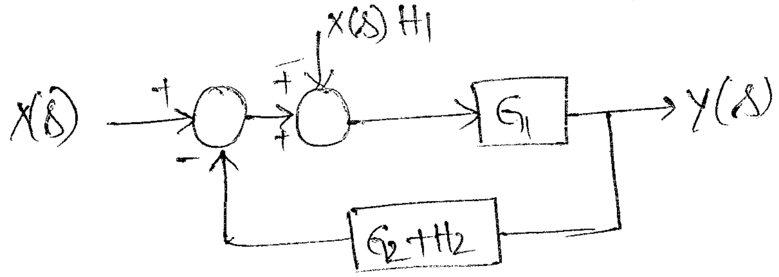
4) 10.44:-



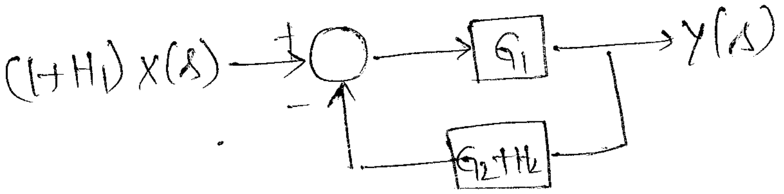
Step 1 :



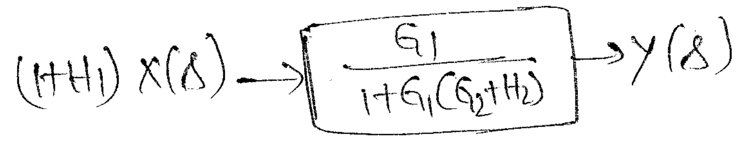
Step 2 :



Step 3 :



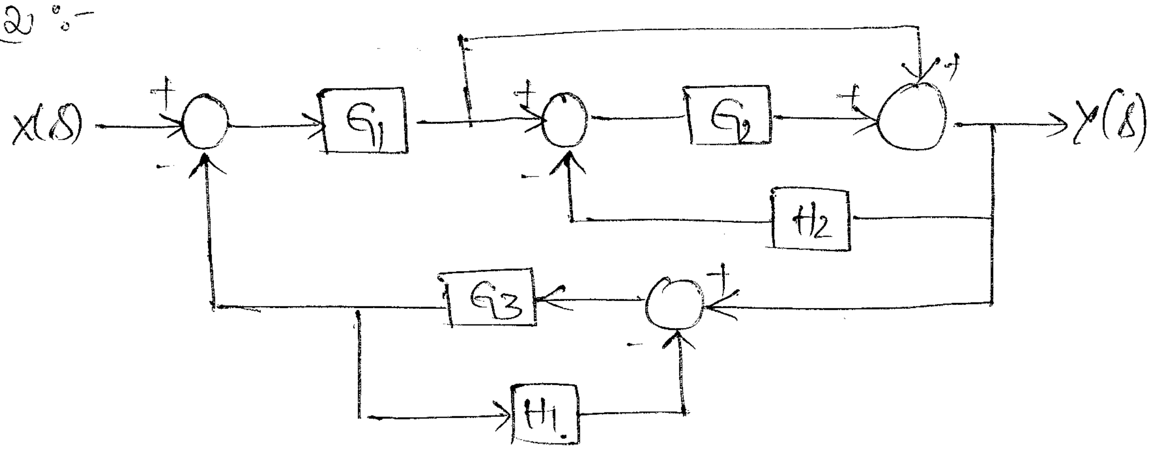
Step 3 :



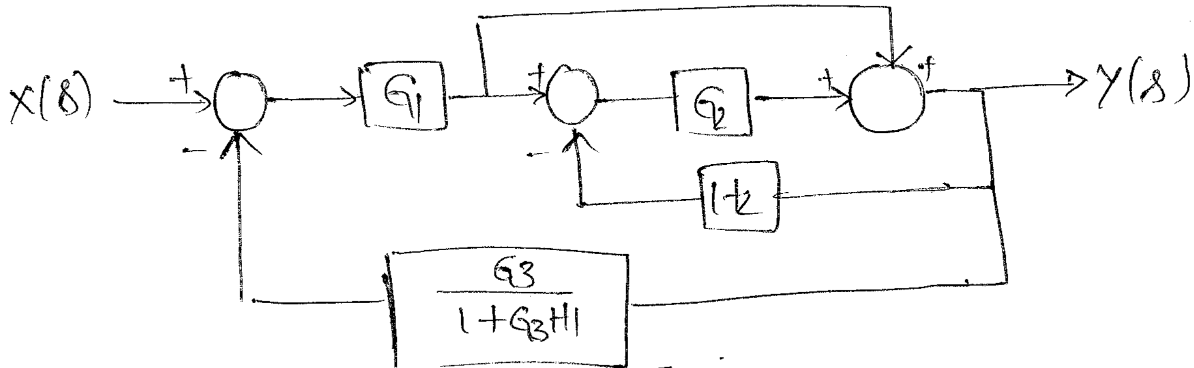
$$\therefore \frac{Y(s)}{(1+H_1)X(s)} = \frac{G_1}{1+G_1(G_2+H_2)}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{G_1(1+H_1)}{1+G_1(G_2+H_2)}$$

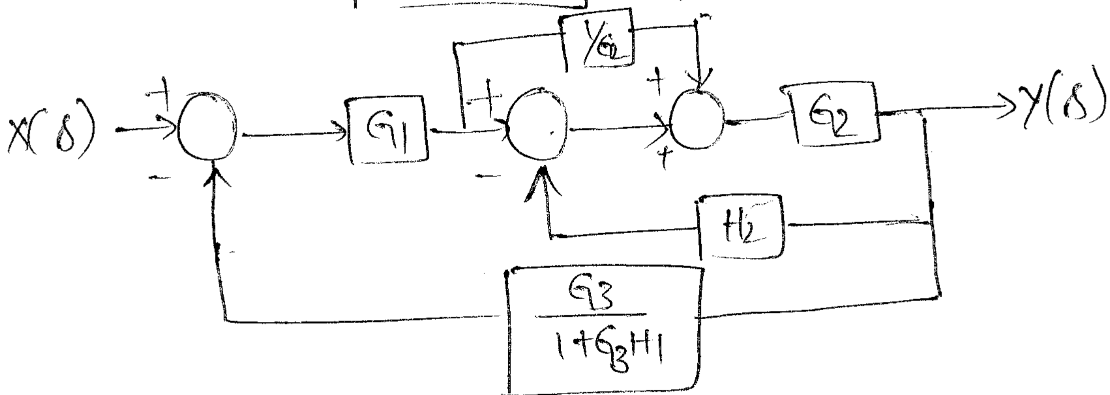
⑤ 10.4.2 :-



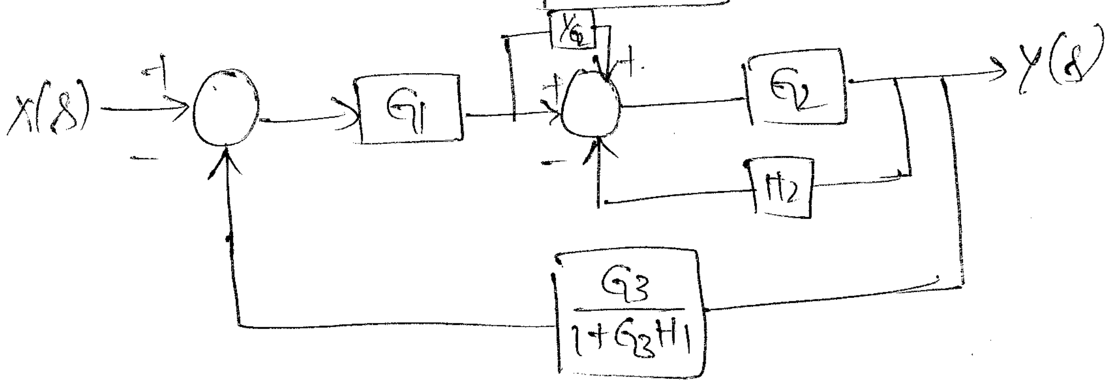
Step ① :



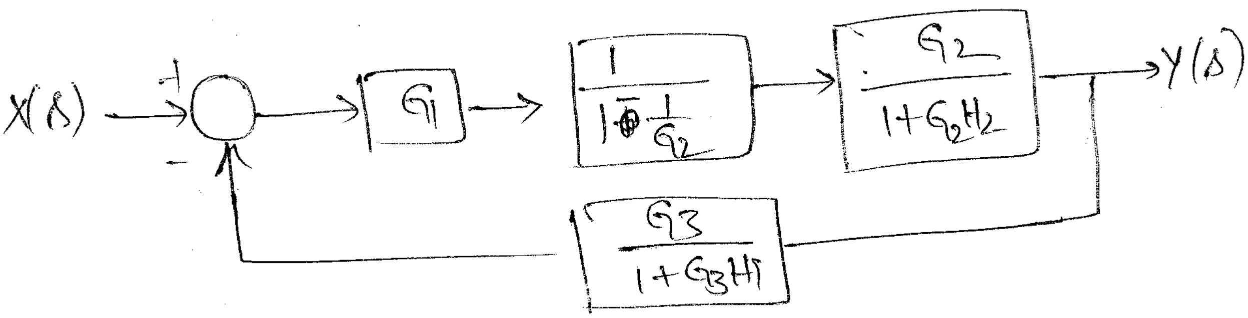
Step ② :



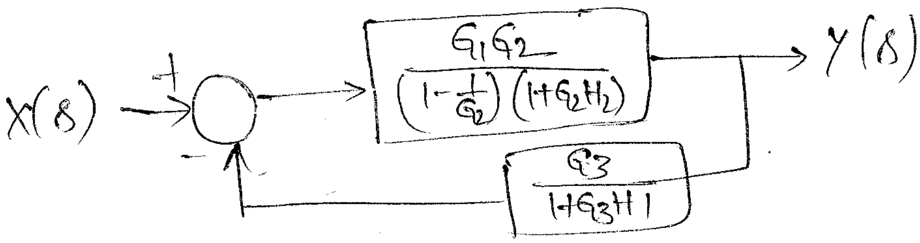
Step ③ :



Step ④ :



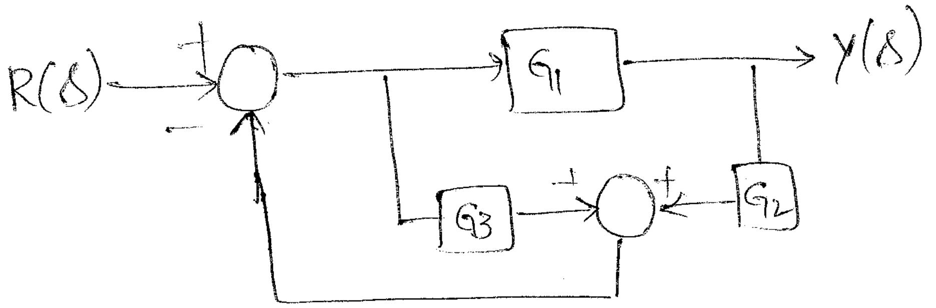
Step ⑤ :



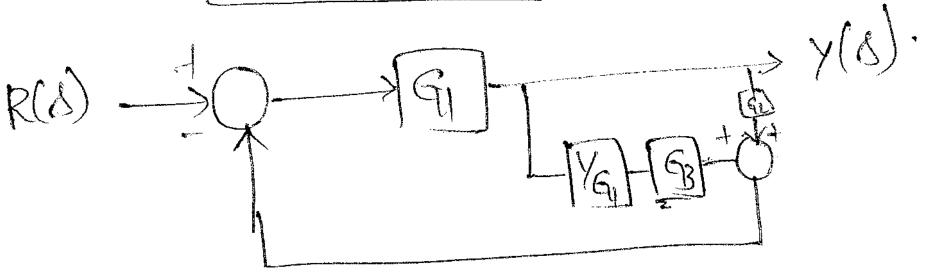
$$\therefore \frac{Y(s)}{X(s)} = \frac{\left(\frac{G_1 G_2^2}{(G_2-1)(1+G_2H_2)}\right)}{1+\left(\frac{G_1G_2^2}{(G_2-1)(1+G_2H_2)}\right)\left(\frac{G_3}{1+G_3H_1}\right)}$$

$$= \frac{G_1G_2^2(1+G_3H_1)}{(G_2-1)(1+G_2H_2)(1+G_3H_1)+G_1G_2^2G_3}$$

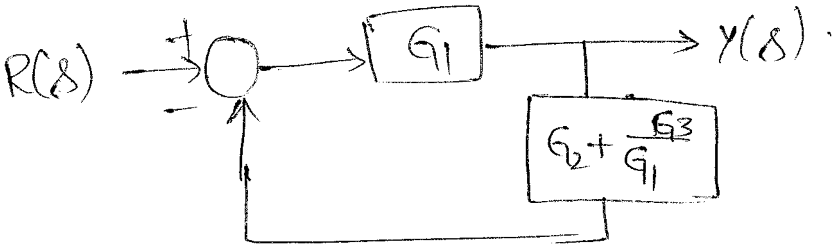
(6) 10.4.8 :-



step ① :-



step ② :-



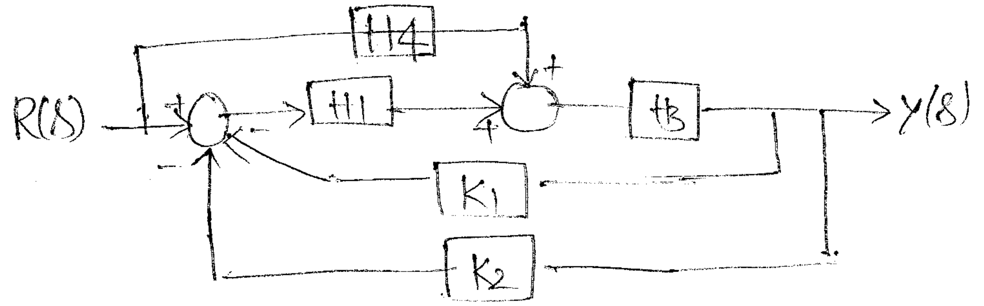
step ③ :-

$$\therefore \frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1\left(G_2+\frac{G_3}{G_1}\right)}$$

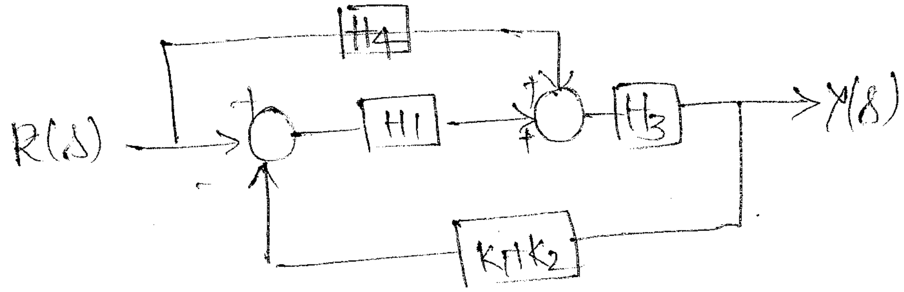
$$= \frac{G_1}{1+(G_1G_2+G_3)}$$

$$= \frac{G_1}{1+G_1G_2+G_3}$$

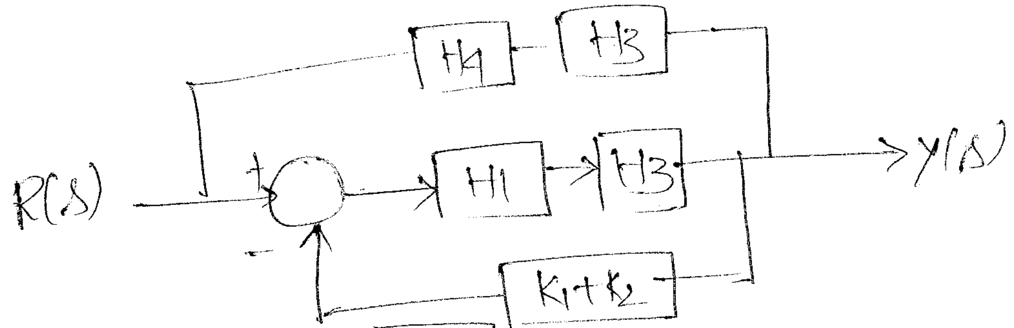
(7) 10.46 :-



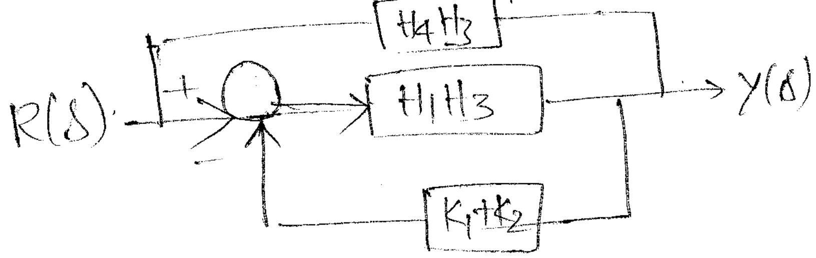
Step (1) :-



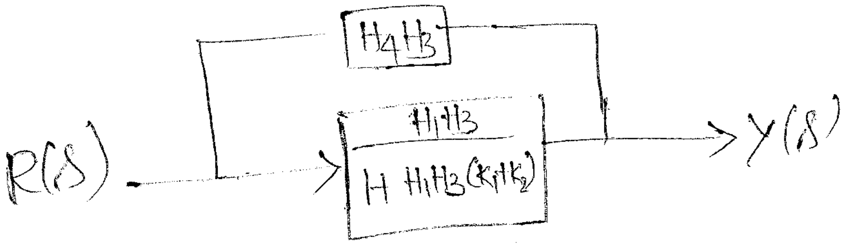
Step (2) :-



Step (3) :

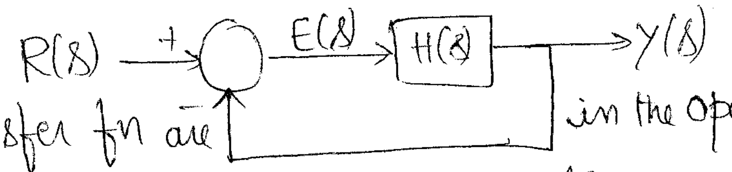


Step (4) :



$$\therefore \frac{Y(s)}{R(s)} = H_4 H_3 + \frac{H_1 H_3}{1 + H_1 H_3 (K_1 + K_2)}$$

(ii). considering the system



All the poles of closed loops transfer fn are in the open LHP then ~~show that~~ find out e(t) as $t \to \infty$ for

a) $\frac{Y(s)}{E(s)} = H(s) = \frac{1}{s}$, if $r(t) = u_s(t)$

b) $\frac{Y(s)}{E(s)} = H(s) = \frac{1}{s^2}$, if $r(t) = r_p(t)$.

$$R(s) \longrightarrow \boxed{\frac{H(s)}{1+H(s)}} \longrightarrow Y(s) \quad \text{and} \quad E(s) = R(s) - Y(s),$$

$$\frac{Y(s)}{R(s)} = \frac{H(s)}{1+H(s)}$$

$$\therefore \quad E(s) = \frac{R(s)}{1+H(s)}.$$

(a) $r(t) = u_s(t) \Rightarrow R(s) = \frac{1}{s}$.

and $H(s) = \frac{1}{s}$.

$$\therefore \quad E(s) = \frac{1/s}{1+1/s} = \frac{1}{s+1}$$

$$\Rightarrow \quad L^{-1}\{E(s)\} = L^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}u_s(t)$$

$$\Rightarrow \quad e(t) = e^{-t}u_s(t)$$

$$\Rightarrow \quad \lim_{t\to\infty} e(t) = \lim_{t\to\infty} e^{-t} = \frac{1}{e^{\infty}} = 0$$

$$\therefore \quad e(t) \longrightarrow 0 \quad \text{as } t \to \infty.$$

(b) $r(t) = r_p(t) \Rightarrow R(s) = \frac{1}{s^2}$

and $H(s) = \frac{1}{s^2}$

$$\therefore \quad E(s) = \frac{1/s^2}{1+1/s^2} = \frac{1}{s^2+1}.$$

$$\Rightarrow \quad L^{-1}\{E(s)\} = L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$\Rightarrow \quad e(t) = L^{-1}\left\{\frac{1/2i}{s-i} - \frac{1/2i}{s+i}\right\}.$$

$$= \frac{1}{2i} L^{-1}\left\{\frac{1}{s-i} - \frac{1}{s+i}\right\}$$

$$= \frac{1}{2i}\left\{e^{it} - e^{-it}\right\}. = \sin t \, u_s(t).$$

$$\Rightarrow \quad \lim_{t\to\infty} e(t) = \frac{1}{2i}\lim_{t\to\infty}\left\{e^{it} - e^{-it}\right\}. = \lim_{t\to\infty} \sin t \, u_s(t)$$

$$= \lim_{t\to\infty} \sin t.$$

$\therefore$ as $t \to \infty$, $e(t)$ is equal to a value with in $[0, 1]$

II 10.21: Consider the transfer functions

$$H_1(s) = \frac{Y_1(s)}{X_2(s)} = \frac{100}{s^2+6s+100}$$

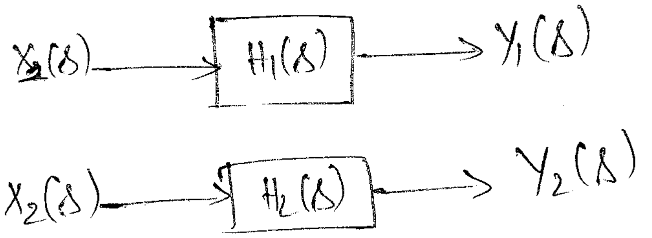
$$H_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{s}{s+20}$$ and $X_1(s)$ is i/p signal.

and $Y_2(s)$ is o/p signal of composite system when they are connected in

a) Cascade interconnection b) parallel interconnection

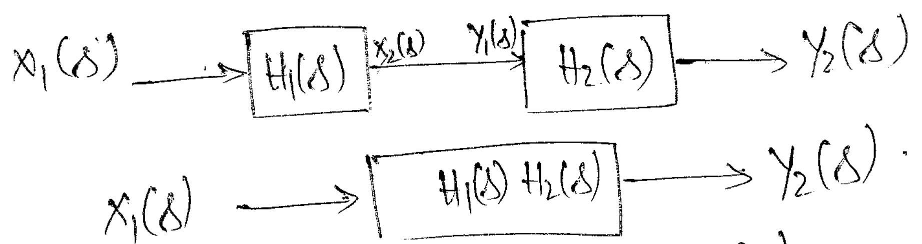
c) feedback configuration [supposing $Y(s)$ is the o/p signal] draw the block diagram & find the transfer function of the composite system

Sol: ~~(a) Cascade~~ Connection



assuming $X_2(s) = Y_1(s)$

(a) Cascade Connection



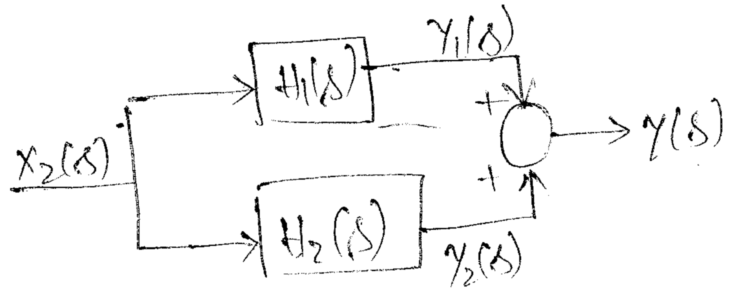
$$\therefore \frac{Y_2(s)}{X_1(s)} = H_1(s)\,H_2(s)$$

$$= \left(\frac{100}{s^2+6s+100}\right)\left(\frac{s}{s+20}\right)$$

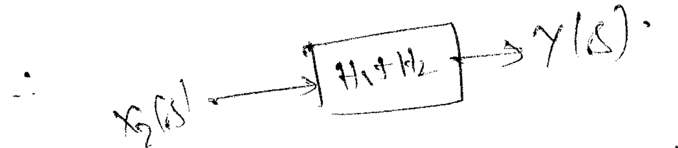
$$= \frac{100s}{s^3+26s^2+220s+2000}$$

(b) parallel connection

assuming
$X_1(s) = X_2(s)$



$\therefore$



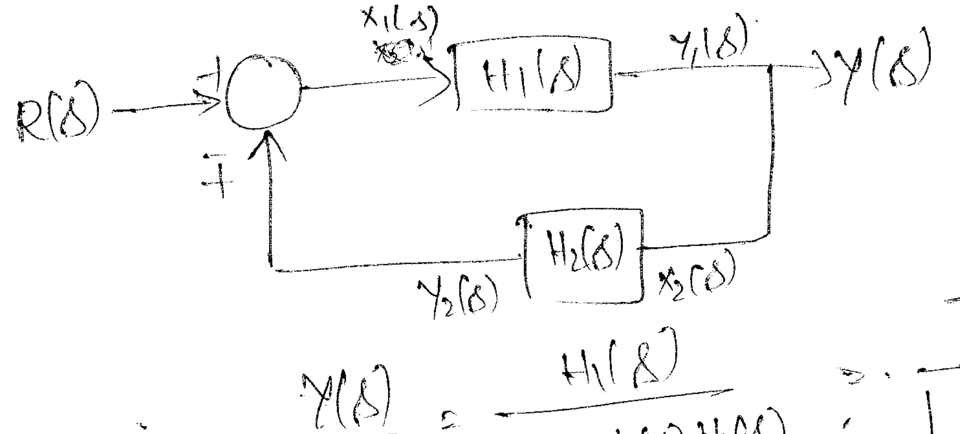
$$\therefore \frac{Y(s)}{X_2(s)} = H_1(s) + H_2(s)$$

$$= \frac{100}{s^2+6s+100} + \frac{s}{s+20}$$

$$= \frac{100s + 2000 + s^3 + 6s^2 + 100s}{s^3 + 6s^2 + 100s + 20s^2 + 120s + 2000}$$

$$= \frac{s^3 + 6s^2 + 200s + 2000}{s^3 + 26s^2 + 220s + 2000}$$

(c) feed back configuration



$$\therefore \frac{Y(s)}{R(s)} = \frac{H_1(s)}{1 \pm H_1(s) H_2(s)} = \frac{\frac{100}{s^2+6s+100}}{1 \pm \frac{100}{s^2+6s+100} \cdot \frac{s}{s+20}}$$

$$= \frac{100s + 2000}{(s^2+6s+100)(s+20) \pm 100s}$$

$$= \frac{100s + 2000}{s^3 + 26s^2 + (220 \pm 100)s + 2000}$$